# Abrupt changes in alpha decay systematics as a manifestation of collective nuclear modes

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An abrupt change in $\alpha$ decay systematics around the $N = 126$ neutron shell closure is discussed. It is explained as a sudden hindrance of the clustering of the nucleons that eventually form the $\alpha$ particle. This is because the clustering induced by the pairing mode acting upon the four nucleons is inhibited if the configuration space does not allow a proper manifestation of the pairing collectivity.

## I. INTRODUCTION

It is nearly a century ago that the Geiger-Nuttall law, which was to revolutionize physics by its implications, was formulated based on $\alpha$ decay systematics [1, 2]. Indeed, its explanation by Gamow [3] and also by Gurney and Condon [4] required to accept the probabilistic interpretation of Quantum Mechanics. The extend to which this was revolutionary can perhaps best be gauged by noticing the multitude of models that have been put forward by outstanding physicists as an alternative to the probabilistic interpretation. This debate rages even at present [5].

The Gamow theory reproduced the Geiger-Nuttall law nicely. One can assert that this is an effective theory, where concepts like "frequency of escape attempts" have to be introduced. Yet Gamow's theory is so successful that even today it is applied, with minor changes, in the studies of radioactive decays (e.g., Refs. [6–8]). In fact, a proper calculation of the decay process needs to address first the clustering of the nucleons at a certain distance outside the nuclear surface and, in a second step, the evaluation of the penetrability through the Coulomb and centrifugal barriers should be performed at the distance where the cluster was formed. The first step is a challenging undertaking because a proper description of the cluster in terms of its components requires a microscopic many-body framework that is very complicated. This is the reason why usually effective approaches are used when dealing with clusterization. That is, one evaluates the penetrability, which is an easy task specially if semi-classical approaches are applied, and free parameters are introduced for the clustering process trying to reproduce experimental data.

One may then wonder why effective approaches have been so successful. The reason is that the $\alpha$-particle formation probability usually varies from nucleus to nucleus much less than the penetrability. In the logarithm scale

of the Geiger-Nuttall law the differences in the formation probabilities are usually small fluctuations along the straight lines predicted by that law [9] for different isotopic chains. The importance of a proper treatment of $\alpha$ decay was attested by a recent calculation which shows that the different lines can be merged in a single line. One thus obtained a generalization of the Geiger-Nuttall law which holds for all isotopic chains and all cluster radioactivities [10, 11]. In this universal decay law (UDL) the penetrability is still a dominant quantity. By using three free parameters only, one finds that all known ground-state to ground-state radioactive decays are explained rather well. This good agreement is a consequence of the smooth transition in the nuclear structure that is often found when going from a nucleus to its neighboring nuclei. This is also the reason why, e.g., the BCS approximation works so well in many nuclear regions.

In this paper we will show that, when a sudden transition occurs in a given chain of nuclei, departures from the UDL can be seen. Our aim is to understand why this difference appears. We will also try to discern whether one can, in general, obtain information about the structure of the nuclei involved in the decay. This would be an important task because many regions of the nuclidic chart now under scrutiny, especially superheavy nuclei, are radioactive and often $\alpha$ decay is the only tool that one has to explore their structure.

In Section II the formation amplitude is defined. In Section III $\alpha$ formation amplitudes extracted from experimental data are presented and abrupt changes are noted. In Section IV the evaluation of the formation amplitudes and half-lives of Po isotopes, which do not follow the UDL, is performed. A summary and conclusions are in Section V.

## II. THE FORMATION AMPLITUDE

After the seminal Gamow's paper, the first attempt to formulate a proper treatment of $\alpha$ decay was based on the compound system theory developed by Teichmann and Wigner [12]. Here the very complicated process occur-

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ring as the compound system decays, is divided into an "internal region", where the compound state is restricted, and the complementary "external region". This division is such that in the external region only the Coulomb and centrifugal forces are important. Thus the decaying system behaves like a two-particle system. This formulation was applied by Thomas to $\alpha$-decay [13] to obtain the classical expression for the decay width $\Gamma_l$ as

$$\Gamma_l(R) = 2\mathcal{P}_l(R)\frac{\hbar^2}{2\mu R}|\mathcal{F}_l(R)|^2, \tag{1}$$

where $l$ is the angular momentum carried by the outgoing $\alpha$-particle, $\mathcal{P}$ is the penetration probability and $\mu$ is the reduced mass corresponding to the final system consisting of an $\alpha$ particle and a daughter nucleus. $R$ is the radius dividing the internal and external regions. At this point the wave function of the $\alpha$-particle already formed in the internal region is matched with the corresponding outgoing two-body wave function in the external region. The amplitude of the wave function in the internal region is the formation amplitude, i.e.,

$$\mathcal{F}_l(R) = \int d\mathbf{R}d\xi_d d\xi_\alpha [\Psi(\xi_d)\phi(\xi_\alpha)Y_l(\mathbf{R})]^*_{J_m M_m}\Psi_m(\xi_d, \xi_\alpha, \mathbf{R}), \tag{2}$$

where $d$, $\alpha$ and $m$ label the daughter, $\alpha$ particle and mother nuclei, respectively. $\Psi$ are the intrinsic wave functions and $\xi$ the corresponding intrinsic coordinates. $\phi(\xi_\alpha)$ is a Gaussian function of the relative coordinates of the two neutrons and two protons that constitute the $\alpha$-particle, coupled to zero angular momentum [14, 15]. The rest of the notation is standard.

One sees from Eq. (2) that $\mathcal{F}_l(R)$ would indeed be the wave function of the outgoing $\alpha$ particle $\psi_\alpha(R)$ if the mother nucleus would behave at the point $R$ as

$$\Psi_m(\xi_d, \xi_\alpha, \mathbf{R}) = [\Psi(\xi_d)\phi(\xi_\alpha)\psi_\alpha(R)Y_l(\mathbf{R})]_{J_m M_m}. \tag{3}$$

Since this is usually a smal component of the mother nucleus wave function, the corresponding formation amplitude (2) is small, of the order of $10^{-2}$ [16]. The main problem in the evaluation of this quantity is the description of the clusterization of the four nucleons that eventually become the $\alpha$-particle. In pursuing this task one has found that the mode that determines clusterization is the pairing vibration [15, 17]. In fact, the study of $\alpha$-clusterization gave rise to the realization that there should be a giant pairing vibration lying high in the nuclear spectra [18, 19]. It is also interesting to notice that the $\alpha$-clusterization in $\alpha$-decaying nuclei has triggered the appearance of effective models where the wave function of nuclei such as $^{212}$Po is assumed to have the form (3). The spectra thus obtained agree well with the corresponding experimental data [20].

Going back to Eq. (1), the wave function corresponding to the external region, i.e., to the outgoing channel, gives rise to the penetration probability $\mathcal{P}_l(R) = kR/(G_l^2 + F_l^2)$, where $G_l$ and $F_l$ are the irregular and

regular Coulomb functions, respectively. From Eq. (1) it is straightforward to see that the width $\Gamma_l(R)$ cannot depend upon R, since outside the range of the nuclear interaction (i.e., just outside the nuclear surface) the internal and external wave functions are the same [21], i.e., $\mathcal{F}_l(R) \propto G_L(R) + iF_l(R)$. This is of course valid provided that the formation amplitude was evaluated properly. In fact a way of probing the calculation is just by investigating whether the width is dependent upon $R$, and in such a case by how much [22].

The $\alpha$-decay half-life can be written as

$$T_{1/2} = \frac{\hbar\ln 2}{\Gamma_l} = \frac{\ln 2}{\nu}\left|\frac{H_l^+(\chi, \rho)}{R\mathcal{F}_l(R)}\right|^2, \tag{4}$$

where $\nu$ is the outgoing velocity of the emitted particle. The distance $R$ will be taken as the touching point, i.e., $R = R_0(A_d^{1/3} + 4^{1/3})$, with $R_0=1.2$ fm. The other quantities are standard, i.e., $H_l^+(\chi, \rho)$ is the Coulomb-Hankel function with arguments $\chi = 4Z_d e^2/\hbar\nu$ and $\rho = \mu\nu R/\hbar$.

In microscopic theories the formation amplitude is evaluated starting from the single-particle degrees of freedom of the neutrons and protons that eventually become the cluster. This requires advanced computing facilities as well as suitable theoretical schemes to describe the clustering process. It is therefore not surprising that the first calculations of absolute decay widths were performed after the appearance of the shell model. These calculations had limited success due to the small shell model spaces that could be included at that time [14]. Yet, in retrospect it is surprising to note the deep insight the pioneers in these shell model calculations had on the role of configuration mixing to induce clustering [23]. That this was indeed the case was shown much later [15, 17] in the case of the decay of the nucleus $^{212}$Po with two protons and two neutrons outside the doubly magic core, $^{208}$Pb, which has been considered as a textbook example in illustrating the clustering and decay of the alpha particle in heavy nuclei (see, e.g., Ref. [24] and references therein). In fact this case is very important for the present paper, since the most significant departure of the UDL from experimental data that we will investigate is in the ground-state to ground-state decays of Po isotopes.

## III. THE EXPERIMENTAL DATA

Using the experimental decay half-lives [25] one can extract the formations amplitudes by applying Eq. (4). One thus obtains

$$\log_{10}|R\mathcal{F}_\alpha(R)| = -\frac{1}{2}\log_{10}T_{1/2}^{\text{Expt.}} + \frac{1}{2}\log_{10}\left[\frac{\ln 2}{\nu}|H_0^+(\chi, \rho)|^2\right]. \tag{5}$$

This is shown in Fig. 1 for different even-even isotopes as a function of the quantity $\rho' = \sqrt{2\mathcal{A}Z_d(A_d^{1/3} + 4^{1/3})}$, where $\mathcal{A} = 4A_d/A_m$. This is one of the two variables that defines the UDL [10].
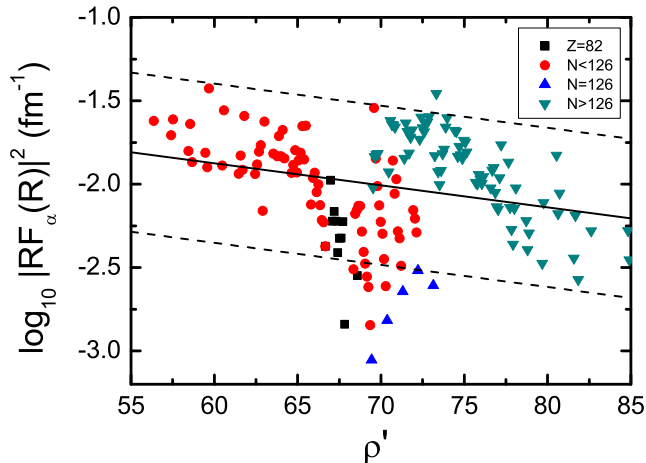
FIG. 1. (Color online) $\log_{10}|RF(R)|^2$ as a function of $\rho'$. The solid line denotes the smooth behavior of the UDL. The values between the two dashed lines differ from the corresponding UDL values by a factor of at most three.

One notices in Fig. 1 that at $\rho' \approx 70$ a division occurs between decays corresponding to $N < 126$ and $N > 126$. Perhaps even more important is that for most cases the UDL predicts the experimental values within a factor of three, except for $N = 126$, where the difference becomes about one order of magnitude. This is so distinct that one may even suspect that the difference in the values of $\mathcal{F}_\alpha$ when going from one nucleus to its neighbors in the vicinity of $N = 126$ overruns the corresponding differences in the penetrability. If one understands the reason of this large variation, $\alpha$ decay may provide a powerful tool to study the structure of decaying nuclei. This point will be analyzed in the next Section.

The case that shows the most significant hindrance corresponds to the $\alpha$ decay of the nucleus $^{210}$Po, with $\log_{10}|R\mathcal{F}_\alpha(R)|^2 < -3$ fm$^{-1}$. The symbols with $\log_{10}|R\mathcal{F}_\alpha(R)|^2 \sim -2.7$ fm$^{-1}$ correspond to the $\alpha$ decays of nuclei $^{208}$Po ($N = 124$), $^{212}$Rn ($N = 126$) and $^{194}$Pb.

Finally, it is worthwhile to point out that this sudden change in $\alpha$-decay systematics at $N = 126$ has also been noticed in Refs. [26–29]. Moreover, in the semiclassical approaches of Refs. [30–36] the decay half-lives of nuclei with $N = 126$ are significantly underestimated. In the alpha-decay formula of Refs. [34, 35], an empirical correction term has been introduced to taken into account the large underestimation around shell closures.

## IV. EVALUATION OF THE HALF-LIVES IN PO ISOTOPES.

In this Section we will analyze, within a microscopic formalism, the half-lives of the isotopes that show the kink at $N = 126$ discussed above. We will take the de-

cay of $^{210}$Po as a typical example and compare it with that of $^{212}$Po. To compare with experimental data, we extract the magnitude of the formation amplitudes from measured half-lives by using Eq. (5). One thus obtains the value $\mathcal{F}_\alpha(R) = 3.305 \times 10^{-3}$ fm$^{-3/2}$ in $^{210}$Po and $\mathcal{F}_\alpha(R) = 1.082 \times 10^{-2}$ fm$^{-3/2}$ in $^{212}$Po, where we used $R = 9.0$ fm. These correspond to a variation in the formation amplitudes by a factor of 3.28, that is a factor of 10.73 in the formation probabilities.

Within the shell model a four-particle state $\alpha_4$ in $^{212}$Po can be written as

$$|^{212}\text{Po}(\alpha_4)\rangle = \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2;\alpha_4)|^{210}\text{Pb}(\alpha_2) \otimes^{210} \text{Po}(\beta_2)\rangle$$

(6)

where $\alpha_2$ ($\beta_2$) labels two-neutron (two-proton) states. For the ground state of $^{212}$Po, it was found [22] that $X(^{210}\text{Pb(gs)} \otimes {}^{210}\text{Po(gs)}) = 0.9$, while $X(^{210}\text{Pb}(2_1^+) \otimes {}^{210}\text{Po}(2_1^+)) = -0.3$.

Each of the terms in Eq. (6) corresponds to neutron-neutron (nn) or proton-proton (pp) states, i.e., states determined by the nn or pp interaction. The neutron-proton (np) interaction mixes those states. In other words, the amplitudes $X$ are influenced by the np interaction. If this interaction is neglected, then only one of the configurations in Eq. (6) would appear. This is done, for instance, in cases where the correlated four-particle state is assumed to be provided by collective vibrational states. Rather typical examples of such states are $|^{210}\text{Pb(gs)}\rangle$ and $|^{210}\text{Po(gs)}\rangle$. It is therefore not surprising that calculations have been performed by assuming that $|^{212}\text{Po(gs)}\rangle$ is a double pairing vibration [15, 37], i.e.,

$$|^{212}\text{Po(gs)}\rangle = |^{210}\text{Pb(gs)} \otimes {}^{210}\text{Po(gs)}\rangle. \quad (7)$$

The corresponding formation amplitude acquires the form,

$$\mathcal{F}_\alpha(R;^{212}\text{Po(gs)}) = \int d\mathbf{R}d\xi_\alpha\phi_\alpha(\xi_\alpha)$$
$$\times \Psi(\mathbf{r_1},\mathbf{r_2};^{210}\text{Pb(gs)})\Psi(\mathbf{r_3},\mathbf{r_4};^{210}\text{Po(gs)}), \quad (8)$$

where $\mathbf{r_1}, \mathbf{r_2}$ ($\mathbf{r_3}, \mathbf{r_4}$) are the neutron (proton) coordinates and $\mathbf{R}$ is the center of mass of the $\alpha$ particle.

With this expression for the formation amplitude the experimental half-life is reproduced rather well if a large number of high-lying configurations is included. These configurations are needed to describe the clusterization between the two neutrons and the two protons in the $\alpha$ particle. Yet the corresponding $\alpha$-decay half-life is still too small by more than one order of magnitude. This is because the neutron-proton interaction is not included in Eq. (7). When this is done, and again a large configurations space is used, the neutrons and protons become also clustered, enhancing the value of the half-life. It is also important to underline that the inclusion of the large configuration space provides a half-life which is independent upon the matching point $R$ [22].

We reproduced these calculations by using a surface delta interaction and nine major shells of a harmonic oscillator (HO) representation. The decay of the nucleus $^{210}$Po(gs) leads to the daughter nucleus $^{206}$Pb(gs), which is a two-hole state. Here we used the five HO major shells corresponding to the single-hole states that describe the wave function of $^{206}$Pb(gs) as

$$|^{206}\text{Pb(gs)}\rangle = \sum_{h_1 \leq h_2} X(h_1 h_2;^{206}\text{Pb(gs)})$$

$$\times \frac{(b_{h_1}^+ b_{h_2}^+)_{0^+}}{\sqrt{2}}|^{208}\text{Pb(gs)}\rangle, \qquad (9)$$

where $h$ labels single-hole states and the hole creation operator is standard, i.e., $b_{jm}^+ = (-1)^{j-m}c_{j-m}$. The formation amplitude becomes,

$$\mathcal{F}_\alpha(R;^{210}\text{Po(gs)}) = \int d\mathbf{R} d\xi_\alpha \phi_\alpha(\xi_\alpha)$$

$$\times \Psi^*(\mathbf{r_1}, \mathbf{r_2};^{206}\text{Pb(gs)})(\mathbf{r_3}, \mathbf{r_4};^{210}\text{Po(gs)}). \qquad (10)$$

By comparing Eqs. (8) and (10) one sees that the only difference between the two expressions is the two-neutron wave function, which corresponds to the two-particle state $^{210}$Pb(gs) in Eq. (8) and to the two-hole state $^{206}$Pb(gs) in Eq. (10). Therefore the kink observed experimentally should be related to the difference in clusterization induced by the pairing force in these two cases. To analyze the clustering features we will consider only the spin-singlet component, i.e., $(\chi_1\chi_2)_0$, of the two-body wave function, since that is the only part entering the intrinsic $\alpha$-particle wave function. This component has the form,

$$\Psi_2(r_1, r_2; \theta_{12}) = \frac{1}{4\pi} \sum_{p \leq q} \sqrt{\frac{2j_p + 1}{2}} X(pq; \text{gs})$$

$$\times \varphi_p(r_1)\varphi_q(r_2)P_{l_p}(\cos\theta_{12}), \qquad (11)$$

where $\varphi$ is the single-particle wave function and $P_l$ is the Legendre polynomial of order $l$ satisfying $P_l(\cos 0) = 1$ (notice that for the ground states studied here it is $l_p = l_q$). As mentioned above, the pairing vibrations show strong clustering features as the number of single-particle states is increased [17]. But another manifestation of the pairing collectivity is an enhancement of the wave function on the nuclear surface. The reason of this enhancement is that all configurations contribute with the same phase in the building up of the two-particle wave function on the nuclear surface. The same mechanism increases the $\alpha$ formation amplitude and, therefore, the relative values of the wave functions of $^{210}$Pb(gs), $^{210}$Po(gs) and $^{206}$Pb(gs) on the nuclear surface give a measure of the importance of the corresponding formation amplitudes.

To study the behavior of the two-particle wave functions we will apply Eq. (11) with $r_1 = r_2$ and $\theta_{12} = 0$. This is reasonable since due to clustering the wave function is strongly peaked at $\theta_{12} = 0$. Calling $R = r_1$, we

have plotted in Fig. 2 $\Psi_2(R, R, 0)$ as a function of $R$. One sees that the wave functions are indeed strongly enhanced at the nuclear surface, as expected. But the important feature for us is that the enhancement is strongest in $^{210}$Pb(gs) and weakest in $^{206}$Pb(gs). This is because there is a relatively small number of configurations in the hole-hole case. In addition, the radial wave functions corresponding to the high-lying particle states extend farther out in space with respect to the hole configurations.
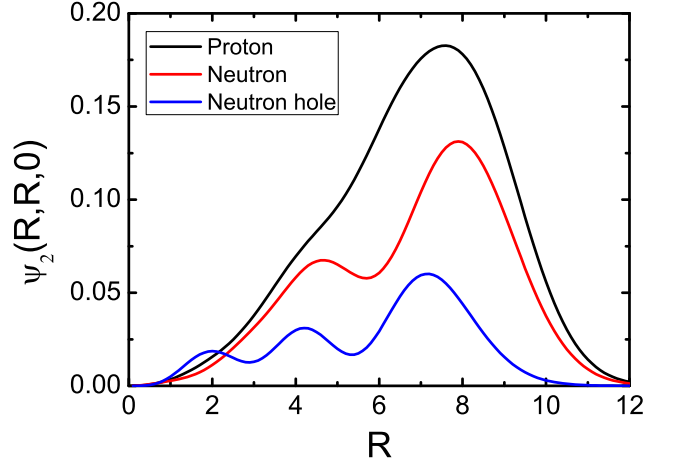


FIG. 2. (Color online) The two-body wave function $\Psi_2$(R,R,0) corresponding to the pairing vibrations in the two-neutron particle $^{210}$Pb(gs), two-proton particle $^{210}$Po(gs) and two-neutron hole $^{206}$Pb(gs) cases.

With these two-body wave functions we proceeded to evaluate the $\alpha$ formation amplitudes in $^{212}$Po(gs) and $^{210}$Po(gs). The results are shown in Fig. 3. One finds from this figure that with $R = 9$ fm the observed ratio between the formation amplitudes in $^{212}$Po and $^{210}$Po can be reproduced nicely.

### A. The neutron-proton interaction

We have assumed (Eq. (7)) that $^{212}$Po(gs) is virtually a correlated two-neutron two-proton state. The same is valid for $^{210}$Po(gs), although here the state is a correlated two-particle (proton) two-hole (neutron) state. This is a manifestation of the pairing vibrational character of two-particle states in the Pb region. That is, the correlated two-particle and two-hole states in the Pb region can be considered as boson degrees of freedom. This was one of the main assumptions in the Nuclear Field Theory [38] as well as in the original Interacting Boson Model [39]. This assumption implies that the neutron-proton interaction does not play a very important role in the spectroscopy of the states. However, as we have seen, this interaction induces the clusterization of neutrons and protons. As pointed out in Ref. [15], in the Pb region low lying
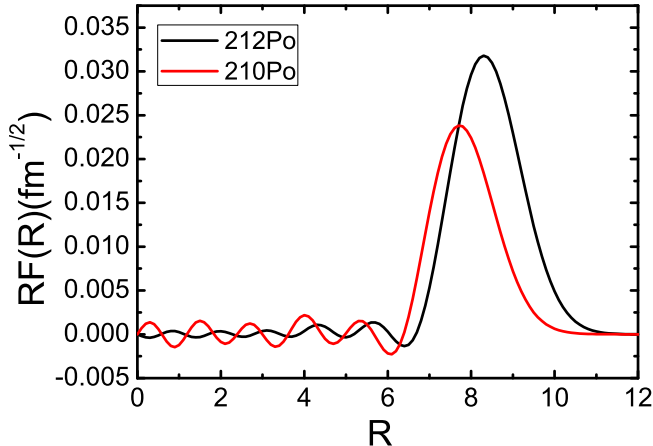
FIG. 3. (Color online) The $\alpha$ formation amplitudes $R\mathcal{F}_\alpha(R)$ corresponding to the nuclei $^{212}$Po(gs) and $^{210}$Po(gs).

neutron and proton single-particle states are very different from each other, or are particle-hole states. Therefore the neutron-proton interaction affects only slightly the ground states and the clusterization occurs through high-lying configurations. This point is supported by our shell-model calculations with the surface delta as well as realistic interactions.

Only when neutrons and protons move in the same orbits it is expected that the neutron-proton interaction would affect significantly the spectroscopic properties as well as the clusterization. We confirmed this by studying a model case in which the core consists of an equal number of neutrons and protons, namely the $\alpha$ decay of the fictional nucleus $^{168}$Po(gs), with two neutrons and two protons outside the core $^{164}$Pb(gs). We used, for neutrons as well as protons, the single-particle states corresponding to protons in the study performed above for $^{210}$Po. We also used the same interaction. As expected, we again found that neutrons and protons are strongly clustered as a result of the corresponding pairing interaction, But also the proton-neutron clustering is significantly enhanced by the proton-neutron interaction, This indicates that in realistic $N = Z$ nuclear regions, for instance around $^{100}$Sn, there should be a large probability to form an alpha particle (see, e.g., Ref. [40]). One can thus conclude that alpha decay probes may be a powerful tool to get information about the structure of heavy $N \approx Z$ nuclei which, otherwise, would be difficult to reach.

## V. SUMMARY AND CONCLUSIONS

In this paper we have applied the recently proposed universal decay law (UDL) [10] to perform a systematic calculation of $\alpha$ decay half-lives over all experimentally known cases. We found that although the UDL reproduces nicely most available experimental data, as expected, there is a case where it fails by a large factor. This corresponds to the $\alpha$ decays of nuclei with neutron numbers equal to or just below $N = 126$. The reason for this large discrepancy is that in $N \leq 126$ nuclei the $\alpha$ formation amplitudes are much smaller than the average quantity predicted by the UDL (Fig. 1). This is an indication that the $\alpha$ decay transitions in these nuclei are hindered with respect to those in the open-shell region.

The case that shows the most significant hindrance corresponds to the $\alpha$ decay of the nucleus $^{210}$Po for which standard shell-model calculation is feasible. Starting from the formal definition of Eq. (2), we calculated the $\alpha$ formation amplitude of $^{210}$Po and compared it with that of $^{212}$Po. In these two cases the formation amplitudes can be described by the simple expressions (8) and (10). We found that the formation amplitude in $^{210}$Po is hindered with respect to the one in $^{212}$Po due to the hole character of the neutron states in the first case. This is a manifestation of the mechanism that induces clusterization, which is favored by the presence of high-lying configurations. Such configurations are more accessible in the neutron-particle case of $^{212}$Po than in the neutron-hole case of $^{210}$Po. This is a general feature in nuclei where neutrons and protons occupy different low-lying major shells. If instead both types of particles occupy the same shells, the neutron-proton interaction is very effective to induce clustering and the formation amplitude increases strongly. This was the case in a calculation that we performed considering the fictitious $N = Z = 84$ $^{168}$Po isotope as the mother nucleus, indicating that even in physically meaningful $N = Z$ nuclear region $\alpha$ decay can be enhanced by large factors.

This allows one to assert that $\alpha$ decay is a powerful tool to investigate the shell structure of very unstable nuclei (including superheavy ones), where often only $\alpha$-decay quantities can be measured.

[1] H. Geiger and J. M. Nuttall, Philos. Mag. **22**, 613 (1911).

[2] H. Geiger, Z. Phys. **8**, 45 (1922).

[3] G. Gamow, Z. Phys. **51**, 204 (1928).

[4] R. W. Gurney and E. U. Condon, Nature **122**, 439 (1928).

[5] G. 't Hooft, arXiv:0908.3408, and references therein.

[6] S. K. Arun, R. K. Gupta, B. B. Singh, S. Kanwar, and M. K. Sharma, Phys. Rev. C **79**, 064616 (2009).

[7] H. F. Zhang, G. Royer, Y. J. Wang, J. M. Dong, W. Zuo, and J. Q. Li, Phys. Rev. C **80**, 057301 (2009).

[8] D. Ni and Z. Ren, Phys. Rev. C **80**, 051303(R) (2009).

[9] B. Buck, A. C. Merchant, and S. M. Perez, Phys. Rev. Lett. **65**, 2975 (1990).

[10] C. Qi, F. R. Xu, R. J. Liotta, and R. Wyss, Phys. Rev. Lett. **103**, 072501 (2009).

[11] C. Qi, F. R. Xu, R. J. Liotta, R. Wyss, M. Y. Zhang, C. Asawatangtrakuldee, and D. Hu, Phys. Rev. C **80**, 044326 (2009).

[12] T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952).

[13] R. G. Thomas, Prog. Theor. Phys. **12**, 253 (1954).

[14] H. J. Mang, Phys. Rev. **119**, 1069 (1960).

[15] I. Tonozuka and A. Arima, Nucl. Phys. **A323**, 45 (1979).

[16] R. G. Lovas, R. J. Liotta, A. Insolia, K. Varga, and D. S. Delion, Phys. Rep. **294**, 265 (1998).

[17] F. A. Janouch and R. J. Liotta, Phys. Rev. C **27**, 896 (1983).

[18] M. W. Herzog, R. J. Liotta and L. J. Sibanda, Phys. Rev. C **31**, 259 (1985).

[19] G. G. Dussel, R. Id Betan, R. J. Liotta and T. Vertse, Phys. Rev. C **80**, 064311 (2009).

[20] B. Buck, A. C. Merchant and S. M. Perez, Phys. Rev. C **51**, 559 (1995).

[21] E. Maglione, L. S. Ferreira and R. J. Liotta, Phys. Rev. Lett. **81**, 538 (1998).

[22] G. Dodig-Crnković, F. A. Janouch, and R. J. Liotta, Nucl. Phys. **A501**, 533 (1989).

[23] J. O. Rasmussen, in K. Siegbahn (Ed.), Alpha- Beta- and Gamma-Ray Spectroscopy, Vol. 1, North-Holland, Amsterdam, 1965, p. 701.

[24] A. Astier, P. Petkov, M.-G. Porquet, D. S. Delion, and P. Schuck, Phys. Rev. Lett. **104**, 042701 (2010).

[25] G. Audi, O. Bersillon, J. Blachol, and A. H. Wapstra, Nucl. Phys. **A729**, 3 (2003).

[26] B. Al-Bataina and J. Jänecke, Phys. Rev. C **37**, 1667 (1988).

[27] K. S. Toth, C. R. Bingham, J. C. Batchelder, L. T. Brown, L. F. Conticchio, C. N. Davids, R. J. Irvine, D. Seweryniak, D. M. Moltz, W. B. Walters, J. Wauters, and E. F. Zganjar, Phys. Rev. C **60**, 011302(R) (1999).

[28] J. Wauters, P. Dendooven, M. Huyse, G. Reusen, P. Van Duppen, P. Lievens, and the ISOLDE Collaboration, Phys. Rev. C **47**, 1447 (1993).

[29] K. Van de Vel, A. N. Andreyev, D. Ackermann, H. J. Boardman, P. Cagarda, J. Gerl, F. P. Heßberger, S. Hofmann, M. Huyse, D. Karlgren, I. Kojouharov, M. Leino, B. Lommel, G. Münzenberg, C. Moore, R. D. Page, S. Saro, P. Van Duppen, and R. Wyss, Phys. Rev. C **68**, 054311 (2003).

[30] B. A. Brown, Phys. Rev. C **46**, 811 (1992).

[31] D. N. Poenaru and M. Ivascu, J. Phys. **44**, 791 (1983).

[32] D. N. Poenaru and W. Greiner, Phys. Scr. **44**, 427 (1991).

[33] D. N. Poenaru, I. H. Plonski, and W. Greiner, Phys. Rev. C **74**, 014312 (2006).

[34] D. N. Poenaru, I. H. Plonski, R. A. Gherghescu, and W. Greiner, J. Phys. G: Nucl. Part. Phys. **32**, 1223 (2006).

[35] D. N. Poenaru, R. A. Gherghescu, and N. Carjan, Eur. Phys. Lett. **77**, 62001 (2007).

[36] M. Ismail, A. Y. Ellithi, M. M. Botros, and A. Adel, Phys. Rev. C **81**, 024602 (2010).

[37] F. A. Janouch and R. J. Liotta, Phys. Rev. C **25**, 2123 (1982).

[38] P. F. Bortignon, R. A. Broglia, D. R. Bes, and R. Liotta, Phys. Rep. **30**, 305 (1977).

[39] A. Arima and F. Iachello, Ann. Rev. Nucl. Part. Sci. **31**, 75 (1981).

[40] S.N. Liddick *et al.*, Phys. Rev. Lett. **97**, 082501 (2006).